Opetopes, rewriting, and koszulity

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Opetopes

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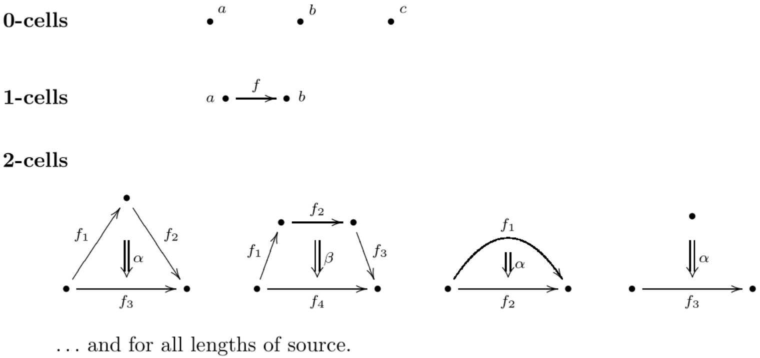
Examples



Higher category theory

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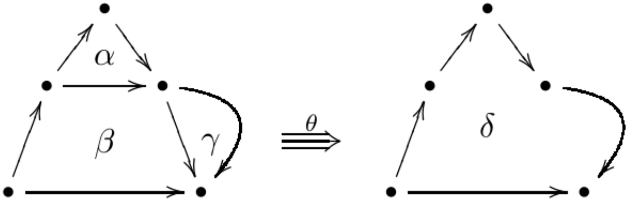

If the pasting diagram (α, β, γ) have a *universal filler* θ, then its target δ is the *weak composite*.

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Theorem

- *The category of n-polygraphs ($n > 2$) is not a presheaf category [Che12].*
- *The category of opetopic sets is equivalent to the category of "many-to-one" polygraphs. (The "many-to-many" aspect of general polygraphs can cause problems for checking confluence)*

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- We want to it lift further to the level of opetopes.

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- Operads encode algebras, opetopes encode operads, opetopes encode opetopes. Opetopic koszulity would be in a sense "most general".

To summarize

- Study higher rewriting **with** opetopes.
- Study rewriting **of** opetopes.
- Develop homological and homotopical machinery.

Thank you for your attention

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